UE Fluids Problem F6 Solution $\qquad$ S'08

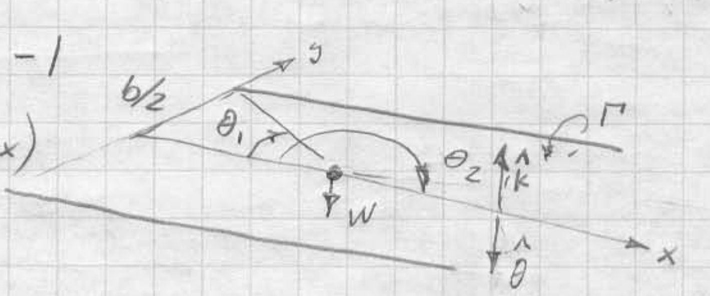
a) From F6 notes, tangential velocity of a straight vortex segment is

$$\vec{V} = \frac{\Gamma\hat{\theta}}{4\pi h}\int_{\theta_1}^{\theta_2} \sin\theta \, d\theta = \frac{\Gamma\hat{\theta}}{4\pi h}\left(-\cos\theta\right|_{\theta_1}^{\theta_2} = \frac{\Gamma\hat{\theta}}{4\pi h}\left(\cos\theta_1 - \cos\theta_2\right)$$

In this case $\hat{\theta} = -\hat{k}$, $-w\hat{k} = 2\vec{V}$ (two vortices contribute)

Also, $\cos\theta_1 = \dfrac{x}{\sqrt{x^2+(b/2)^2}}$, $\cos\theta_2 = -1$

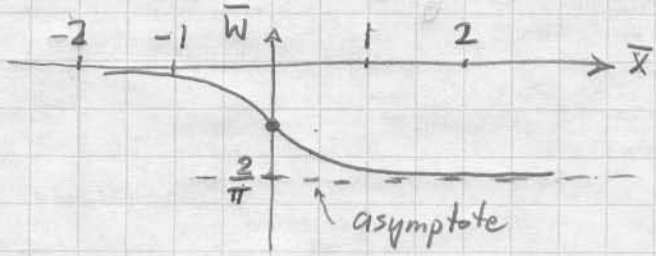
and $h = b/2$ (distance from vortex)



$$\therefore \quad w = \frac{-\Gamma}{4\pi(b/2)}\left(\frac{x}{\sqrt{x^2+(b/2)^2}} + 1\right)\cdot 2$$

or $$w = -\frac{\Gamma}{\pi b}\left(\frac{x}{\sqrt{x^2+(b/2)^2}} + 1\right)$$

or $$\bar{w} \equiv \frac{wb}{\Gamma} = \frac{-1}{\pi}\left(\frac{\bar{x}}{\sqrt{\bar{x}^2+1/4}} + 1\right)$$



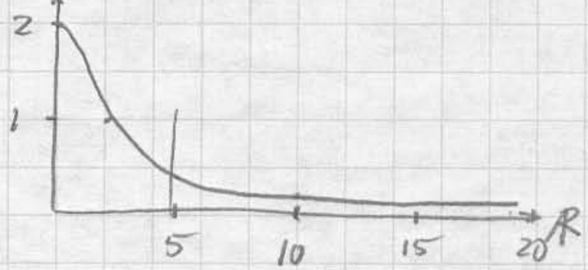
b) $AR = \frac{b}{c}$, at $x = \pm c/2$, $\bar{x} = \pm\frac{x}{b} = \pm\frac{c}{2b} = \pm\frac{1}{2}\frac{1}{AR}$

$\bar{w}(0) = \frac{-1}{\pi}$

$$\therefore \quad \frac{\bar{w}(+1/2AR) - \bar{w}(-1/2AR)}{-1/\pi} = \frac{1/2AR - (-1/2AR)}{\sqrt{(1/2AR)^2 + 1/4}} = \frac{2}{\sqrt{1+AR^2}}$$

c) The relative change in w

gets reasonably small for $AR > 5$ or so



L.L. Theory may be questionable

for fighter wings with smaller AR's than 5 or so.